2.153 Adaptive Control

2019 Problem Set #3

Out: Wednesday Oct 2
Due: Oct 11 5pm, to jegaudio-at-mit.edu

In all problems, provide technical justifications for your answers.

Problem 1 (6-1)

pg. 269 If θ is a constant vector in $\mathbb{R}^n$, and the functions $u : \mathbb{R}^+ \to \mathbb{R}^n$ and $y : \mathbb{R}^+ \to \mathbb{R}$ are related by

$$u^\top(t)\theta = y(t)$$

determine the conditions on u so that θ can be determined from input-output data. Can θ be estimated exactly in finite time?

Problem 2 (6-2)

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(a) If in the equation

$$\dot{x}(t) = -u(t)u^\top(t)x(t) \qquad x, u : \mathbb{R}^+ \to \mathbb{R}^n$$

$u \notin \Omega_{(n,T)}$, can you characterize the asymptotic behavior of $x(t)$?

(b) In part (1), if $n = 2$ and

$$u(t) = \left[c_1 \sin T + e^{-t}, c_2 \sin t + e^{-t}\right]^\top,$$

where c_1 and c_2 are constants, show that $c_2x_1 - c_1x_2$ does not tend to zero asymptotically, where $x = [x_1, x_2]^\top$.

Problem 3 (6-3)

pg. 269 If $u : \mathbb{R}^+ \to \mathbb{R}^2$, show that $u = [\sin T, \cos t]^\top \in \Omega_{(2,T)}$, while $u = [\sin\sqrt{t}, \cos\sqrt{t}]^\top \notin \Omega_{(2,T)}$. Provide a qualitative justification for your answer.

Problem 4 (6-6)

pg. 270 If $r(t) = \sum_{i=1}^{n} a_i \sin \omega_i t$, where $a_i \neq 0$, $\omega_i \neq \omega_j$, for $i \neq j$, and $i, j = 1, \ldots, n$, show that the state x of a $2n^{\text{th}}$ order system $\dot{x} = Ax + br$ where A is asymptotically stable and (A, b) is controllable belongs to $\Omega_{(2n,T)}$.

Problem 5 (6-9)

pg. 270 In Problem 1, if $u(t) \equiv 0$ for $t > t_0 + T$ and

$$\int_{t_0}^{t_0+T} u(\tau)u^\top(\tau)d\tau = Q > 0$$

can you suggest an adaptive algorithm for determining the estimate $\widehat{\theta}$ of θ such that $\lim_{t\to\infty}[\widehat{\theta}(t) - \theta] = 0$?

Problem 6 (4-2)

pg. 178 Determine an adaptive observer for identifying the unknown parameter vectors a and b in the system: $\dot{x} = Ax + bu$; $y = h^\top x$ where $a = [a_1, \ldots, a_n]^\top$,

$$A = \left[\begin{array}{c|c} \text{-a} & I \\ & 0 \end{array}\right]$$

and $h^\top = [1, 0, \ldots, 0]$.