2.153 Adaptive Control

2019 Problem Set #1

Out: Wednesday Sep 11
Due: Friday Sep 20 5pm, 3-348

Six problems assigned: Chapter 1, Problem 5; Chapter 3, Problems 2, 3, 4, 5, 6.

Problem 1: (1-5)

pg. 37 Some simple feedback systems are described below. In each case, discuss whether or not you would consider the system to be adaptive. Give reasons for your answer. If you decide that it is not adaptive, indicate the modifications in the problem that would require an adaptive solution. Once again, give reasons why you consider the solution to be adaptive.

(a) A water filtration and purification system is shown in Fig. 1 Water from a reservoir A is admitted through a valve v_1 into tank B where it is filtered. The tank B is connected through a valve v_2 to chemical and biochemical purifiers downstream. The objective is to control the flow through the system by adjusting v_1 and v_2 so that the level of the water in B is regulated around a fixed value.
When the resistance in B is a constant, fixed openings of v_1 and v_2 are found to be adequate. However, when the resistance of the filtering tank changes due to the deposition of impurities, the valves v_1 and v_2 have to be adjusted continuously.

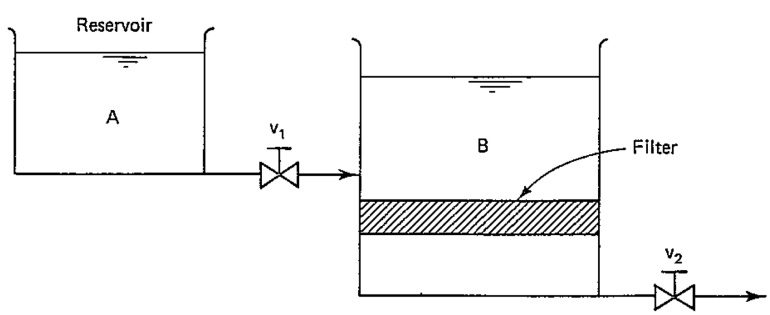

Figure 1: Water filtration system.

(b) A turbo generator shown in Fig 2, consisting of an air pressure turbine that drives a synchronous generator, is used to keep the output voltage and frequency at constant values. The actuating valve of the turbine has a nonlinear characteristic, and the relationship between the field current and the voltage shows a strong nonlinear behavior. Denoting the valve position and the field current by u_1 and u_2 respectively, and the output frequency f and voltage v by y_1 and y_2 respectively, a two-input two-output dynamical system can be described. When the load R_L varies, the turbo generator works in different operating points resulting in significant

variations in frequency and voltage. Hence, the valve position u_1 and field current u_2 have to be adjusted continuously.

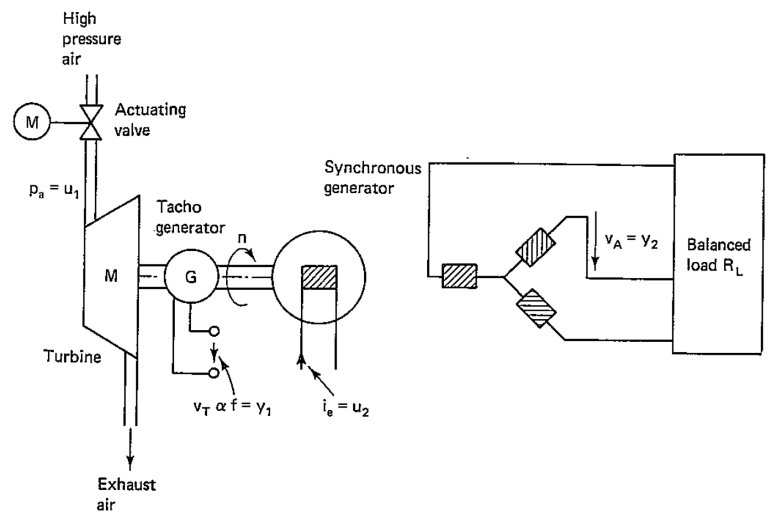

Figure 2: Turbo generator system.

(c) The object of control in paper machines is to adjust the control variables u_1 and u_2 in such a manner as to minimize the deviations of the basis weight y_1 and moisture y_2 of the manufactured paper from the predetermined values. The variables u_1 and u_2 correspond to the thick stock valve and steam pressure valve in the drier section respectively (Fig 3). u_1 and u_2 must be adjusted when the quality of the paper changes due to moisture deviations, pressure changes in the head box, or due to varying conditions in the drying section.

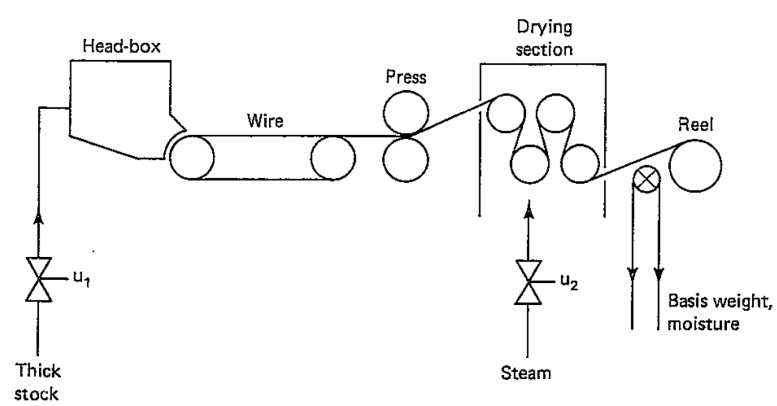

Figure 3: Control of a paper machine. Courtesy of Pergamon Press.

(d) A simple model of an aircraft system can be described by the differential equation

$$\ddot{x} + 2[\zeta(t) + k(t)]\dot{x} + x = r(t)$$

where $r(t)$ is a reference input. The desired response of the system is achieved when the damping factor $\zeta(t) + k(t)$ is a constant and equals 0.7. It is known that $\zeta(t)$ varies with

time due to changes in air density. The step response of the system is determined periodically (by using an additive step input) and the damping parameter is estimated. This in turn is used to adjust the velocity feedback gain k so that the estimate of the damping factor of the overall system is 0.7. Determine an adaptive law for adjusting $k(t)$ so that it can reach 0.7. Under what conditions do you expect the system above with your adaptive law to perform satisfactorily?

Problem 2: (3-2)

pg. 136 If an asymptotically stable nonlinear plant is described by the scalar differential equation

$$\dot{x}_p = \sum_{i=1}^{N} a_i f_i(x_p, u, t)$$

where f_i are known bounded functions for $i = 1, \ldots, N$, so that x_p is bounded for a bounded input u, and $u(t)$ and $x_p(t)$ can be measured at each instant, describe a method of estimating the unknown parameters $a_1, \ldots, a_N$. Use the adaptive identification methods (either error model 1 or 3) if possible. Explain its stability properties.

Problem 3: (3-3)

pg. 136 If ϕ is the parameter error vector, the methods described in this chapter result in

$$\lim_{t\to\infty} \dot{\phi}(t) = 0$$

Does this imply that $\phi(t)$ tends to a constant? Justify your answer with a proof if the answer is yes, and a counterexample if it is no.

Problem 4: (3-4)

pg. 137 When an identification or an adaptive control problem yields an error differential equation

$$\dot{e} = -e + \phi x_p$$

the adaptive law $\dot{\phi} = -ex_p$ was chosen in Chapter 3. This makes $V(e, \phi) = \frac{1}{2}e^2 + \frac{1}{2}\phi^2$ a Lyapunov function since $V > 0$ and $\dot{V} = -e^2 \leq 0$. If the adaptive law is

$$\dot{\phi} = -\phi - ex_p$$

$V(e, \phi)$ would still be a Lyapunov function with $\dot{V} = -\phi^2 - e^2 < 0$ assuring uniform asymptotic stability in the large of the origin in the (e, ϕ) space. Why then is such an adaptive law not used?

Problem 5: (3-5)

pg. 137 If a nonlinear plant is described by the differential equation

$$\dot{x}_p = \sum_{i=1}^{N} a_i f_i(x_p) + b_p u$$

where $a_1, \ldots, a_n$ and b are unknown, but $\text{sgn}(b_p)$ is known, $f_1(\cdot), \ldots, f_n(\cdot)$ are known bounded functions, and x_p can be measured at each instant, design a controller to match x_p to the output of a time-invariant reference model which is asymptotically stable.

Problem 6: (3-6)

pg. 137 Let the error differential equation in an adaptive problem be $\dot{e} = -e + \phi x_p$. If the parameter error ϕ is adjusted using the adaptive law $\dot{\phi} = -\gamma(t) e x_p$, where $\gamma(t)$ is a time-varying gain, what are the conditions on $\gamma(t)$ so that the origin in the (e, ϕ) space is uniformly stable?