2.153 Adaptive Control
2015 Problem Set #3

Out: Wednesday March 5
Due: Friday March 13 at 3-441 by 11:59pm

Problem 1
Simulate the scalar adaptive system below for the different values $\gamma$ in your designed adaptive law when $a_m = -1$, $k_m = 1$, $a_p = 1$, and $k_p = 2$.

\[
\dot{x}_p = a_p x_p + k_p u \\
\dot{x}_m = a_m x_m + k_m r
\]

Problem 2
Simulate the “revised design” adaptive PI controller for the DC motor as given in Lecture 6. Use $J = 2$ and $B = 0.5$. Show the responses for different $\hat{J}(0)$ and $\hat{B}(0)$ with the goal of tracking a smoothened step input. You can pick any amplitude you prefer. The adaptation gains $\gamma_1$ and $\gamma_2$ can be arbitrary.

Problem 3
Simulate an adaptive state-feedback controller for the following system

\[
\dot{x} = Ax + B\Lambda u \\
\dot{x} = A_m x_m + B_m r
\]

where $A$ and $B$ are known, and $\Lambda$ is unknown, given by

\[
A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 1 \\ 7 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \quad \Lambda = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}
\]

and different $r$, (i) $r = [5, 5]^T$, (ii) $r = [5 \sin(\omega t), 5 \sin(\omega t)]^T$. You can pick any Hurwitz $A_m$ and $B_m$ that satisfies the matching conditions specified in class.

Problem 4:
Consider the plant given in Figure 1 below, a DC Motor, with $\tau$ as the control input and $x$ as the position output. All parameters are unknown.

![Figure 1: DC motor for problem 1](image-url)
(a) Design an adaptive PID controller so that $x \to x_d$, where $x_d$ is a desired position signal.

- You can assume that $k_p, J_p$ are positive.
- You can assume that $\dot{x}$ is measurable.

(b) Design an adaptive phase-lead compensator so that $x \to x_d$, where $x_d$ is a desired position signal.

- You can assume that $k_p, J_p$ are positive.
- You can assume that $\dot{x}$ is measurable.

In both cases (a) and (b) above, show that such a $\tau$ exists (ie. algebraic part has a solution). Also show that the closed-loop adaptive system has bounded solutions, and that asymptotic tracking can be achieved. Clearly state any other assumptions you need to make.