2.153 Adaptive Control
Review

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Adaptive Systems;
Adaptive Systems; History;

\[ y = \theta^T u; \]

Dynamic Systems
\[ \dot{x} = a_p x + k_p u, \quad a_p, k_p \text{ unknown} \]

Control:
\[ u = \theta x + kr, \quad \dot{\theta} = -\text{sign}(k)(x - x_m), \quad \dot{k} = -\text{sign}(k)(x - x_m) r \]

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Adaptive Systems; History; Control of Plants with Unknown Parameters

\[ \dot{x} = a_p x + k_p u, \quad a_p, k_p \text{ unknown} \]

\[ u = \theta x + kr, \quad \dot{\theta} = -\text{sign}(k)(x - x_m) x, \quad \dot{k} = -\text{sign}(k)(x - x_m) r \]
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems $y = \theta^T u$; Identify $\theta$
Adaptive Systems; History; Control of Plants with Unknown Parameters
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Dynamic Systems
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems $y = \theta^T u$; Identify $\theta$

Dynamic Systems $\dot{x} = a_p x + k_p u$, $a_p, k_p$ unknown
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems \( y = \theta^T u \); Identify \( \theta \)

Dynamic Systems \( \dot{x} = a_p x + k_p u \), \( a_p, k_p \) unknown
- Identify \( a, b \)
- Control: \( u = \theta x + kr \), \( \dot{\theta} = -\text{sign}(k)(x - x_m)x \), \( \dot{k} = -\text{sign}(k)(x - x_m)r \)
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems $y = \theta^T u$; Identify $\theta$

Dynamic Systems $\dot{x} = a_p x + k_p u$, $a_p, k_p$ unknown
- Identify $a, b$
- Control: $u = \theta x + kr$, $\dot{\theta} = -\text{sign}(k)(x - x_m)x$, $\dot{k} = -\text{sign}(k)(x - x_m)r$
Adaptive Control: ORM

\[ u(t) = \theta x_p + kr \]
\[ \dot{\theta}(t) = -\gamma \text{sign}(k_p) e x_p, \quad \dot{k}(t) = -\gamma \text{sign}(k_p) e r \]

(i) Stability; (ii) \( \lim_{t \to \infty} e(t) = 0 \)
Dynamic Systems $\dot{X} = A_pX + B_pu$. Find $u$ so that $X \to X_m$
Dynamic Systems $\dot{X} = A_p X + B_p u$. Find $u$ so that $X \to X_m$

- $B_p$ known
- $B_p = B \Lambda$, $B$ known, $\Lambda$ diagonal, sign of entries known.
- $B_p$ unknown: Local stability
$b_p k^* = b_m \quad A_p + b_p \theta^{*\top} = A_m$

\[
\dot{\theta} = -\text{sign}(k^*) e^{\top} P b_m X_p \\
\dot{k} = -\text{sign}(k^*) e^{\top} P b_m r
\]
Adaptive Control of \( n \)th order plants - with multiple inputs; \( B_p \) known

Plant: \[ \dot{X}_p = A_pX_p + B_pu \]

Choose Controller: \[ u = \Theta_A(t)X_p + \Theta_B^*r \]

Closed-loop: \[ \dot{X}_p = [A_p + B_p\Theta_A(t)]X_p + B_p\Theta_B^*r \]

\[ A_p + B_p\Theta_A^* = A_m; \quad B_p\Theta_B^* = B_m, \quad \tilde{\Theta}_A = \Theta_A - \Theta_A^* \]

\[ \dot{X}_p = A_mX_p + B_p\tilde{\Theta}_A X_p + B_m r \]

Reference Model \[ \dot{X}_m = A_mX_m + B_m r \]

Error Model \[ \dot{e} = A_me + B_p\tilde{\Theta}_A X_p \]

Use Error Model 2 analysis
Adaptive Control of $n$th order plants - with multiple inputs; $B_p = B\Lambda$

Plant: $\dot{X}_p = A_p X_p + B\Lambda u$

$A_p, \Lambda$ unknown, $B$ known, $\Lambda \in \mathbb{R}^{m \times m}$ and diagonal known

Matching Conditions: $A_m = A_p + B\Lambda \Theta_A^*, B\Lambda \Theta_B^* = B_m$

Reference Model

Controller: $u = \Theta_A(t)X_p + \Theta_B(t)r$

Choose $\dot{\Theta}_A = -\Gamma \text{sign}(\Lambda)B^T Pe X_p^T$, $\Gamma = \Gamma^T > 0$

Choose $\dot{\Theta}_B = -\Gamma \text{sign}(\Lambda)B^T Pe r^T$

$\Rightarrow$ Stability. $\lim_{t \to \infty} e(t) = 0$. 

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Adaptive Control of $n$th order plants - with multiple inputs; General $B_p$

Plant: $\dot{X}_p = A_p X_p + B_p u$

$A_p, B_p$ unknown

Matching Conditions: $A_m = A_p + B_m \Theta^*_A$, $B_p \Theta^*_B = B_m$

Reference Model

Controller: $u = \Theta_B(t) (\Theta_A(t) X_p + r)$

Error equation: $\dot{e} = A_m e + B_m \left( \tilde{\Theta}_A X_p + \tilde{\Psi} u \right)$

$\tilde{\Theta}_A = \Theta_A - \Theta_A^*$

$\tilde{\Psi} = \Theta_B^{*-1} - \Theta_B^{-1}$

Leads to Local Stability.