2.153 Adaptive Control
Lecture 10
Uniform Asymptotic Stability

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- **Pset #1** out: Thu 19-Feb, **due: Fri 27-Feb**
- **Pset #2** out: Wed 25-Feb, **due: Fri 6-Mar**
- **Pset #3** out: Wed 4-Mar, **due: Fri 13-Mar**
- **Pset #4** out: Wed 11-Mar, **due: Fri 20-Mar**
- **Midterm (take home)** out: Mon 30-Mar, **due: Fri 3-Apr**
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems

\[ y = \theta T u \]

Dynamic Systems

\[ \dot{x} = a_p x + k_p u, \quad a_p, k_p \text{unknown} \]

Identify \( a \), \( b \)

Control:

\[ u = \theta x + kr, \quad \dot{\theta} = -\text{sign}(k)(x - x_m) \]
\[ \dot{k} = -\text{sign}(k)(x - x_m) r \]

ORM:

\( x_m \) determined by a linear reference model

CRM:

\( x_m \) determined by a closed-loop reference model

n-th order Dynamic Systems

\[ \dot{X} = A_p X + B_p u \]

Find \( u \) so that \( X \to X_m \)

\( B_p \) known

\( B_p = B \Lambda \), \( B \) known, \( \Lambda \) diagonal, sign of entries known.

\( B_p \) unknown: Local stability

Adaptive PI Control

Adaptive PID Control

Adaptive Phase Lead Compensators

Both \( x \) and \( \dot{x} \) measurable

Only \( x \) measurable
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems \( y = \theta^T u \); Identify \( \theta \)

Dynamic Systems \( \dot{x} = a_p x + k_p u, \ a_p, k_p \) unknown
- Identify \( a, b \)
- Control: \( u = \theta x + kr, \ \dot{\theta} = -\text{sign}(k)(x - x_m)x, \ \dot{k} = -\text{sign}(k)(x - x_m)r \)
- ORM: \( x_m \) determined by a linear reference model
- CRM: \( x_m \) determined by a closed-loop reference model
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems $y = \theta^T u$; Identify $\theta$

Dynamic Systems $\dot{x} = a_p x + k_p u$, $a_p, k_p$ unknown
- Identify $a, b$
- Control: $u = \theta x + kr$, $\dot{\theta} = -\text{sign}(k)(x - x_m)x$, $\dot{k} = -\text{sign}(k)(x - x_m)r$
- ORM: $x_m$ determined by a linear reference model
- CRM: $x_m$ determined by a closed-loop reference model

$n$th order Dynamic Systems $\dot{X} = A_p X + B_p u$. Find $u$ so that $X \rightarrow X_m$
- $B_p$ known
- $B_p = B\Lambda$, $B$ known, $\Lambda$ diagonal, sign of entries known.
- $B_p$ unknown: Local stability
Adaptive Systems; History; Control of Plants with Unknown Parameters

Algebraic Systems \( y = \theta^T u \); Identify \( \theta \)

Dynamic Systems \( \dot{x} = a_p x + k_p u, \) \( a_p, k_p \) unknown

- Identify \( a, b \)
- Control: \( u = \theta x + kr, \dot{\theta} = -\text{sign}(k)(x - x_m)x, \dot{k} = -\text{sign}(k)(x - x_m)r \)
- ORM: \( x_m \) determined by a linear reference model
- CRM: \( x_m \) determined by a closed-loop reference model

\( n \)th order Dynamic Systems \( \dot{X} = A_p X + B_p u. \) Find \( u \) so that \( X \rightarrow X_m \)

- \( B_p \) known
- \( B_p = B\Lambda, B \) known, \( \Lambda \) diagonal, sign of entries known.
- \( B_p \) unknown: Local stability

Adaptive PI Control

Adaptive PID Control

Adaptive Phase Lead Compensators

- Both \( x \) and \( \dot{x} \) measurable
- Only \( x \) measurable
Stability and Asymptotic Stability

(i)

\[ V(x) > 0 \]
\[ \dot{V}(x) \leq 0 \]

\[ x \in \mathcal{L}_\infty \quad \text{— Uniform Stability} \]
Stability and Asymptotic Stability

(i)

\[ V(x) > 0 \]
\[ \dot{V}(x) \leq 0 \]

\[ \Rightarrow x \in \mathcal{L}_\infty \quad - \text{Uniform Stability} \]

(ii)

\[ V(x) > 0 \]
\[ \dot{V}(x) < 0 \]

\[ \Rightarrow x \in \mathcal{L}_\infty, \quad \lim_{t \to \infty} x(t) = 0 \quad - \text{Uniform Asymptotic Stability} \]
Stability and Asymptotic Stability

(i)

\[ V(x) > 0 \]
\[ \dot{V}(x) \leq 0 \]
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(iii) Adaptive Systems:

\[ V(x) > 0 \]
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Stability and Asymptotic Stability

(i)  
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  - Uniform Stability

(ii)  
\[ V(x) > 0 \]
\[ \dot{V}(x) < 0 \]
\[ \Rightarrow x \in \mathcal{L}_\infty, \lim_{t \to \infty} x(t) = 0 \]  
  - Uniform Asymptotic Stability

(iii) Adaptive Systems:  
\[ V(x) > 0 \]
\[ \dot{V}(x) \leq 0 \]
\[ \Rightarrow x \in \mathcal{L}_\infty. \text{ How do we show that } \lim_{t \to \infty} x(t) = 0 \]
Example: First-order plant - Adaptive Control

\[ u(t) = \theta x_p + kr \]

\[ \dot{\theta}(t) = -\gamma \text{sign}(k_p) e \]

(i) Stability; (ii) \( \lim_{t \to \infty} e(t) = 0 \)
Example: First-order plant - Adaptive Control

\[ u(t) = \theta x_p + kr \]
\[ \dot{\theta}(t) = -\gamma \text{sign}(k_p) e_p, \quad \dot{k}(t) = -\gamma \text{sign}(k_p) e_r \]

(i) Stability; (ii) \( \lim_{t \to \infty} e(t) = 0 \) (iii) When do \( \theta(t) \to \theta^* \) and \( k(t) \to k^* \)?
Example: First-order plant - Adaptive Control

\[ u(t) = \theta x_p + kr \]
\[ \dot{\theta}(t) = -\gamma \text{sign}(k_p) e x_p, \quad \dot{k}(t) = -\gamma \text{sign}(k_p) e r \]

(i) Stability; (ii) \( \lim_{t \to \infty} e(t) = 0 \) (iii) When do \( \theta(t) \to \theta^* \) and \( k(t) \to k^* \) ?

\[ a_p + k_p \theta^* = a_m, \quad k_p k^* = k_m \]