2.153 Adaptive Control
Lecture 1
Simple Adaptive Systems: Identification

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Parameter Adaptation - Recursive Schemes

Adaptive Control: The control of Uncertain Systems

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Adaptive Control (in this Course):
The control of Linear Time-invariant Plants with Unknown Parameters

Adaptive Control: A Parametric Framework

- Nonlinear, time-varying, with unknown parameter θ

$$\dot{x} = f(x, u, \theta, t) \qquad y = h(x, u, \theta, t)$$

- Linear Time-Varying (LTV) with unknown parameter θ

$$\dot{x} = A(\theta, t)x + B(\theta, t)u \qquad y = C(\theta, t)x + D(\theta, t)u$$

- Linear Time-Invariant (LTI) with unknown parameter θ

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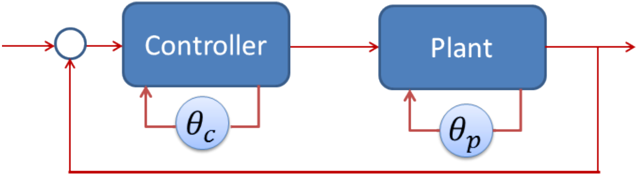
Controlled System (closed-loop): System

Direct and Indirect Adaptive Control



θ_p: Plant parameter - unknown;

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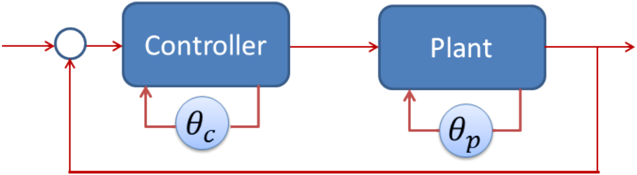

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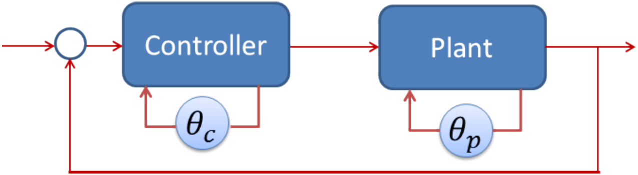
Indirect Adaptive Control: Estimate θ_p as $\hat{\theta}_p$. Compute $\hat{\theta}_c$ using $\hat{\theta}_p$.

$$\theta_p \rightarrow \widehat{\theta}_p \rightarrow \widehat{\theta}_c$$

Direct and Indirect Adaptive Control



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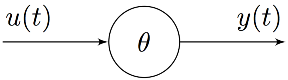
Indirect Adaptive Control: Estimate θ_p as $\hat{\theta}_p$. Compute $\hat{\theta}_c$ using $\hat{\theta}_p$.

$$\theta_p \to \widehat{\theta}_p \to \widehat{\theta}_c$$

Direct Adaptive Control: Directly estimate θ_c as $\hat{\theta}_c$. Compute the plant estimate $\hat{\theta}_p$ using $\hat{\theta}_c$

$$\theta_p \to \theta_c \to \widehat{\theta}_c$$

Identification of a Single Parameter



θ: Unknown, scalar

$$y(t) = \theta u(t)$$

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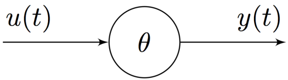

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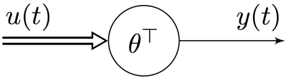
$$y(t) = \theta u(t)$$

Identify θ using measurements $\{u(t), y(t)\}$.

Identification of a Vector Parameter



$$y(t) = \theta^T u(t)$$

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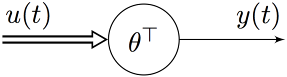

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$y \in \mathbb{R}$,

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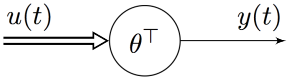

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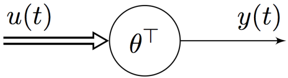

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$y \in \mathbb{R}$, $\theta \in \mathbb{R}^n$, $u : \mathbb{R}^+ \to \mathbb{R}^n$

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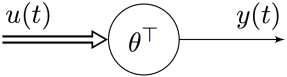

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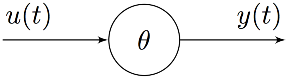

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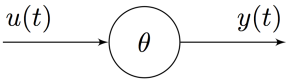

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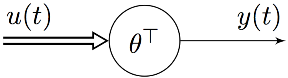

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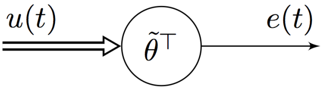
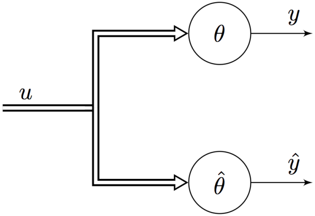
Identify θ as $\widehat{\theta}(t)$ at every instant

Error Model 1

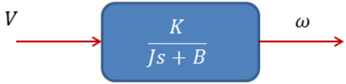




$\widehat{\theta}$: Unknown, $u(t)$ and $e(t)$ can be measured at each instant t.

Identification of a Parameter in a Dynamic System

Simplest Transfer Function of a Motor:



V : Voltage input $\qquad$ ω: Angular Velocity output

K, J, B: Physical parameters

Plant:

$$\frac{K}{Js+B} = \frac{a_1}{s+\theta_1}$$

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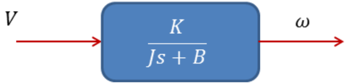

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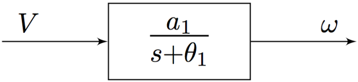
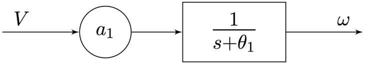
K, J, B unknown $\Rightarrow a_1,\ \theta_1$ unknown

One way of identifying parameters a_1 and θ_1





Assume that a_1 is known.

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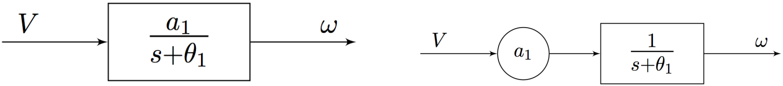

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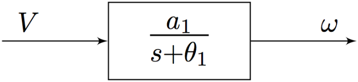
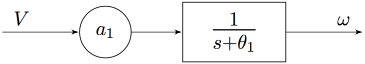


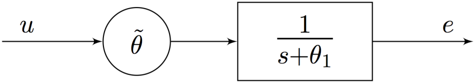


Assume that a_1 is known. Identify θ_1 as $\widehat{\theta}$. $\widetilde{\theta} = \widehat{\theta} - \theta_1$

Plant: $\dot{\omega} = -\theta_1 \omega + u \qquad u = a_1 V$

Error Model 3

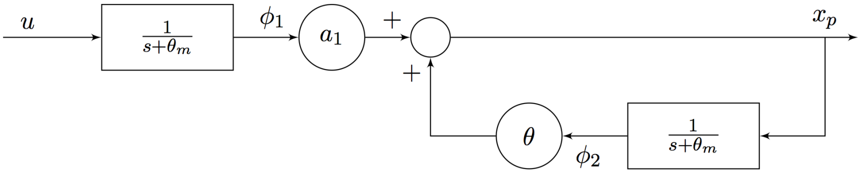
$$\dot{e} = -\theta_1 e + \widetilde{\theta} u$$



An alternate procedure for identifying θ_1:

$$\frac{a_1}{s+\theta_1} = \frac{\dfrac{a_1}{s+\theta_m}}{1+\dfrac{\theta_m-\theta_1}{s+\theta_m}}$$

$\theta \equiv \theta_1 - \theta_m$



Stability

Behavior near an Equilirbium Point.

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Consider the following dynamical system

$$\begin{aligned} \dot{x}(t) &= f(x(t), t) \\ x(t_0) &= x_0 \end{aligned} \tag{1}$$

Definition: equilibrium point (pg 45) The state x_{eq} is an *equilibrium point* of (1) if it satisfies:

$$f(x_{\text{eq}}, t) = 0 \tag{2}$$

for all $t \geq t_0$.

Stability of LTI Plants

A motivating example: determine the stability of the origin for the following scalar system

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Can determine the stability of the origin by evaluating eigenvalues of A

$$x(t) = e^{A(t-t_0)}x(t_0)$$

$$A = V\Lambda V^{-1};\ \ V : \text{from eigenvector};\ \ \Lambda : diag(\lambda_i) : \text{from eigenvalues}$$

Stability follows if $Re(\lambda_i) \leq 0$
Asymptotic stability follows if $Re(\lambda_i) < 0$.

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Lyapunov's methods allow us to determine the stability of an equilibrium for such a system without solving the differential equation!

Lyapunov Stability

For the system

$$\dot{x} = f(x)$$

Let

- (i) $V(x) > 0, \ \forall x \neq 0, \text{and } V(0) = 0$
  - (ii) $\dot{V}(x) = \left(\frac{\partial V}{\partial x}\right)^T f(x) < 0$
  - (ii) $V(x) \to \infty \text{ as } \|x\| \to \infty$

Then $x = 0$ is asymptotically stable.

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If instead of (ii), we have

- (ii') $\dot{V} \leq 0$

Then $x = 0$ is stable.

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Error Model 1 leads to the following

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$$\begin{aligned} V &= \frac{1}{2}x^T x \\ \dot{V} &= x^T A(t)x = -x^T u(t)u^T(t)x = -\left(x^T u(t)\right)^2 \leq 0 \end{aligned}$$

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We therefore conclude that error model 1 leads to a stable parameter estimation. Asymptotic stability will be shown later.