Adaptive Control of Hypersonic Vehicles in Presence of Aerodynamic and Center of Gravity Uncertainties

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Abstract—The paper proposes a new class of adaptive controllers that are suited for command tracking in hypersonic vehicles (HSV) in the presence of aerodynamic and center of gravity (CG) uncertainties under cruise conditions. This class pertains to linear plants whose states are accessible with a partially known input matrix $B$. It is well known that standard multivariable adaptive controllers only yield local stability when the input matrix is completely unknown. In this paper, it is shown that when additional information regarding the structure of $B$ is available, this difficulty can be overcome using the proposed class of controllers. In addition, a nonlinear damping term is added to the adaptive law to further improve the stability characteristics. The proposed adaptive controller is shown to be applicable for command following in HSV when subjected to a class of aerodynamic and center of gravity uncertainties. A model that accurately captures the effect of CG shifts on the longitudinal dynamics of a HSV is derived from first principles. Performance improvements are shown using simulation studies carried out on a full scale nonlinear model of the HSV. It is shown that the adaptive controller can tolerate larger CG shifts as compared to a fixed gain controller while tracking reference commands in velocity and altitude.

I. INTRODUCTION

There has been a renewed interest in the design and development of hypersonic cruise vehicles as they provide a reliable and cost-effective access to space. The control design for such a vehicle has been a challenge as the dynamics of the Hypersonic Vehicle (HSV) comprise strong coupling between aerodynamic, propulsion and structural effects [1-4]. Given that there is limited wind-tunnel data, the environment is harsh and uncertain, the operating conditions are large and varying, the models may be poorly known, and the actuators are subject to various anomalies including saturation, an adaptive controller that can cope with many of these uncertainties is desirable [4,5,7,10]. Various uncertainties have been addressed in the context of the HSV, which include geometric and inertial [6], [7], aerodynamic [4], [5], inertial elastic [9], and thrust uncertainties [10]. In this paper, we propose a control design that expands the class of uncertainties further, and considers aerodynamic uncertainties that occur due to changes in lift and moment co-efficients and center of gravity movements.

The integration of the propulsion system with the vehicle and the interactions between the internal and external flow fields makes characterization of the aerodynamics difficult. Even though large amount of wind tunnel data for the HSV is available, it might not be accurate. An additional class of uncertainties is due to center of gravity (CG) movements, which may directly impact the irregular short-period mode. Since the conservation equations of the HSV are derived by evaluating the moments and forces about the CG, even the linearized equations of motion get altered when the CG moves. Control action under a CG shift can excite flexible dynamics and can introduce instabilities.

The starting point for the controller proposed in this paper is the representation of the aerodynamic and center of gravity uncertainties mentioned above as a class of parametric uncertainties in the underlying linear plant-model. As will be shown in Section II, despite the availability of all states of this model for control, existing results in multivariable adaptive control are inapplicable. As a result, a new controller is derived and shown to globally stabilize the linear plant for this class of parametric uncertainties. In Section V, a linearized plant-model of the HSV is derived, and shown to belong to this class when aerodynamic uncertainties and CG movements are present. The adaptive controller proposed is validated using a simulation of the nonlinear model of the HSV including flexible states used in [4]. The results show the advantage of adaptation compared to a baseline, non-adaptive controller, for a range of CG movements.

The new adaptive controller proposed consists of two novel extensions compared to the standard multivariable adaptive controllers that are well known in the literature [11]. The first is the consideration of uncertainties in the input matrix $B$. It is well known that when $B$ is known or when $B$ includes only scaled-uncertainties [14], [15], a globally stabilizing adaptive controller can be constructed, and that for a general unknown $B$, only local results are available [13]. In this paper, the former class is expanded further by simplifying the control structure of the latter for a class of uncertainties in $B$. The second innovation introduced in the proposed adaptive controller is the use of nonlinear damping. Employed in the past for addressing difficulties introduced due to relative degree [11], the introduction of nonlinear damping here is shown to result in a stable controller and in a better performance in the simulation studies.

II. PROBLEM STATEMENT

Consider the MIMO plant with dynamics,

$$\dot{x} = Ax + Bu$$  (1)
where the state \( x \in \mathbb{R}^{n \times 1} \) is accessible and \( u \in \mathbb{R}^{m \times 1} \) is the control input. The problem is to determine \( u \) when \( A \) and \( B \) are unknown so that the closed-loop system is stabilized and the state \( x \) is brought to zero. This problem has been studied extensively [11] and several global and local results are available. Since these results are pertinent to the contribution of this paper, we briefly review the relevant results in this area.

A. \( B \) known

The simplest adaptive controller that can be derived when states are accessible corresponds to the case when \( A \) is unknown in (1) but \( B \) is completely known. Under the assumption that there exists a \( \theta^* \) such that,

\[
A + B\theta^* = A_m
\]  
(2)

where \( A_m \) is a known Hurwitz matrix, it can be easily shown that the following adaptive controller leads to global stability.

\[
\begin{align*}
\dot{\theta} &= -\Gamma B^T \dot{E} x^T \\
e &= x - x_m \quad \dot{x}_m = A_m x_m
\end{align*}
\]  
(3)

where \( P \) satisfies the Lyapunov equation

\[
A_m^T P + PA_m = -Q < 0.
\]  
(4)

The corresponding Lyapunov function in this case is of the form

\[
V = e^T Pe + Trace(\tilde{\theta}^T \Gamma^{-1} \tilde{\theta})
\]  
(5)

which has a derivative

\[
\dot{V} = -e^T Q e \leq 0
\]  
(6)

where \( \tilde{\theta} = \theta - \theta^* \).

Remark 1: A relaxation of the assumption in (2) has been used extensively in the design of adaptive flight control systems [14]. This corresponds to an assumption that \( B \) is unknown but is such that

\[
B \Lambda^* = B_m
\]  
(7)

where \( B_m \) is known and \( \Lambda^* \) is a diagonal matrix with the signs of its diagonal elements known. Such an assumption is quite reasonable in problems where uncertainties occur due to anomalies in actuators [14],[15].

B. \( B \) unknown

When \( B \) is unknown, the stability result that can be obtained differs drastically from that in the above section [11]. The controller in this case is of the form

\[
\begin{align*}
u &= K \theta x \\
\dot{\theta} &= -\Gamma B_m^T \dot{E} x^T
\end{align*}
\]  
(8)

where \( e \) and \( x_m \) are defined as in (5) and \( P \) satisfies (6). Due to the structure of the control input in (10), assumption (2) is modified as

\[
A + BK^* \theta^* = A_m
\]  
(9)

Since \( B \) is unknown, an additional assumption that a non-singular matrix \( K^* \) exists such that

\[
BK^* = B_m
\]  
(10)

is needed here, where \( B_m \) is known. The main point to note here is that in this case, the stability result is local. This occurs because the underlying error equation is of the form

\[
\dot{e} = A_m e + B_m \tilde{\theta}(t) x + B_m \tilde{\Psi}(t) u
\]  
(11)

which in turn causes the Lyapunov function to be of the form,

\[
V = e^T Pe + Trace(\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \tilde{\Psi}^T \Gamma^{-1} \tilde{\Psi})
\]  
(12)

with a time-derivative,

\[
\dot{V} = -e^T Q e \leq 0
\]  
(13)

We assume that further information is available about \( \Psi^* \) as follows: Let \( \Psi^* \) be such that there exists a sign-definite matrix \( M \) and a known symmetric positive-definite matrix \( \Gamma_0 \) such that

\[
\Gamma_0 \Psi^* + \Psi^* \Gamma_0 = M.
\]  
(14)

Equation (19) implies that \( B_m \) lies in the subspace spanned by the columns of \( B \). Equation (20) essentially implies that partial information is available regarding \( B \) such that a known \( B_m \) and a \( \Psi^* \) that satisfies a sign-definite condition exist. It should be noted that the class of such \( B \)s satisfying (20) is quite larger than those satisfying (9). The assumption in (20) essentially allows us to find a globally stabilizing adaptive controller by not introducing the time-varying parameter \( K \) in (10).

Remark 2: It is easy to show that assumption (20) is satisfied if the unknown \( B \) is of the form

\[
B = \Lambda B_p
\]  
(15)

where \( B_p \in \mathbb{R}^{n \times m}, n \geq m \) is known and is full rank, \( \Lambda \) is unknown and \( \Lambda \) and \( B_p \) satisfy either of the following conditions,

(i) \( \Lambda + \Lambda^T \) is sign-definite

(ii) \( B_p^T \Lambda B_p \) is sign-definite
In both cases, it is straightforward to show that an $M$ exists that satisfies (20) if $B_m = B_p$ and $\Gamma_0 = I$.

**Remark 3:** Again, it can be easily shown that assumption (20) is satisfied if the unknown $B$ is of the of the form

$$B = B_p \Lambda$$

(22)

and the unknown $\Lambda$ satisfies

- symmetric part of $\Lambda$ is sign-definite

In this case an $M$ that satisfies (20) exists if $B_m = B_p$ and $\Gamma_0 = I$.

In the following, we show that if $B$ is unknown but satisfies (19) and (20), a globally stabilizing adaptive controller can be derived.

We choose an adaptive controller of the form,

$$\dot{u} = \theta(t) x$$

$$\dot{\theta} = -GB_p^T P \phi x$$

(23)

leading to the error equations,

$$\dot{e} = A_m e + B_m \Psi^T \dot{\theta} x$$

(26)

$$\dot{\theta} = -G B_m^T P \phi x^T$$

(27)

A Lyapunov function candidate,

$$V = e^T P e + Trace(\dot{\theta}^T (\Psi^T \Gamma) \dot{\theta})$$

(28)

results in

$$\dot{V} = -e^T Q e$$

(29)

using assumption (20). This implies that the closed loop system is globally stable. A straightforward application of Barbalat’s lemma results in,

$$\lim_{t \to \infty} e(t) = \lim_{t \to \infty} x(t) = 0$$

(30)

**Remark 4:** All the above discussions can be extended to the tracking problem where $x_m$ is realized using a reference command $r$ as

$$\dot{x}_m = A_m x_m + B_m r$$

(31)

by modifying $u$ as

$$u = \theta(t) x + N(t) r$$

(32)

and suitably adaptive $N$.

IV. APPLICATION TO HYPERSONIC VEHICLE

In this section we apply the controller design developed in section III to the hypersonic vehicle subjected to aerodynamic and center of gravity uncertainties.

**A. Hypersonic Vehicle Modeling**

The equations of motion of the HSV consist of 5 rigid states, $x = [V, \alpha, Q, h, \theta]^T$ and and 6 elastic states $\eta = [\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6]^T$. The control inputs are elevator deflection ($\delta_e$) and the equivalence ratio ($\phi$) of the scramjet engine. The combined equations of motion are (please see [4] for details),

$$\dot{V} = (T \cos \alpha - D)/m - g \sin(\theta - \alpha)$$

(33)

$$\dot{\alpha} = -(T \sin \alpha + L)/mV + Q + (g/mV) \cos(\theta - \alpha)$$

(34)

$$\dot{\dot{\theta}} = Q$$

(35)

$$\dot{\dot{\eta}_i} = -2 \zeta \omega \dot{\eta}_i - \omega^2 \eta_i + N_i, \quad i = 1, 2, 3$$

(36)

$$\dot{\dot{\gamma}} = Q \dot{\dot{\theta}}$$

(37)

$$\dot{\dot{\phi}} = Q \dot{\dot{\theta}}$$

(38)

$$L = L(x, \eta, \delta_e), \quad D = D(x, \eta, \delta_e), \quad M = M(x, \eta, \delta_e)$$

$$N = N(x, \eta), \quad T = T(x, \eta, \phi)$$

**B. Center of Gravity Modeling**

Assuming that the HSV is a slender symmetric body, the lateral dynamics and the lateral shifts in CG are negligible. Starting from the derivations in [8] and introducing the longitudinal dynamics assumptions, the generalized equations of motion of longitudinal dynamics can be shown to be,

$$\Sigma F_x = m [\ddot{U} + QW - Q^2 \Delta x + \dot{Q} \Delta z + g \sin(\theta)]$$

(39)

$$\Sigma F_y = 0$$

(40)

$$\Sigma F_z = m [W - QU + \dot{Q} \Delta x - Q^2 \Delta z - g \cos(\theta)]$$

(41)

$$\Sigma M_y = I_{yy} \dot{\dot{Q}} + m [(QU - \dot{W} + g \cos(\theta)) \Delta x + (QW + \dot{U} + g \sin(\theta)) \Delta z]$$

(42)

where, $\Delta x$, $\Delta y$ and $\Delta z$ denote the location of CG with respect to an arbitrary body fixed point. Fig 1 shows the body fixed axis system of the HSV. We perform a stability axis transformation to develop a control oriented model. It should be noted that the angle of attack is defined to be the angle that the velocity vector of the CG makes with the fuselage reference line. Under the longitudinal dynamics assumption, the velocity at the center of mass can be shown to be,

$$V_{cg} = (U + Q \Delta z) \hat{i} + (W - Q \Delta x) \hat{k}$$

(43)
and the angle of attack, by definition is

\[ \tan \alpha = \frac{W - Q \Delta x}{U + Q \Delta z} \]  

(44)

If we define the total velocity at the center of mass as \( V \), it can be shown that the following relations hold true,

\[
\dot{V} = (T \cos \alpha - D)/m - g \sin(\theta - \alpha)
\]  

(45)

\[
\dot{\alpha} = -(T \sin \alpha + L)/mV + Q + (g/mV) \cos(\theta - \alpha)
\]  

(46)

It should be noted that (45) and (46) are exactly same as (33) and (34), as though there has been no CG shift. However, the velocity predicted by these equations of motion is not that of body fixed point, but that of the new center of mass. If the moment equation is expressed in terms of the stability axes variables, then (35) gets modified to,

\[
(I_{yy} - m(\Delta x^2 + \Delta z^2))\dot{\phi} = M
\]  

(47)

\[
-m[V(\Delta z \cos \alpha - \Delta x \sin \alpha) - V\dot{\alpha}(\Delta x \cos \alpha + \Delta z \sin \alpha)]
- mg(\Delta x \cos \alpha + \Delta z \sin \alpha)
\]

The above result is reasonable as a shift in the center of gravity does not change the lift produced but only brings about a change in the pitching moment. It should also be noted that a shift in CG changes the moment at trim.

V. LINEARIZATION

A. Linearized Model of the HSV

The design model described in (33)-(37) can be expressed compactly as a non-linear model,

\[ \dot{X} = f(X, U) \]  

(48)

where \( X = [V, \alpha, Q, h, \theta]^T \) is the state vector and \( U = [\phi, \delta_c]^T \) is the control input. We linearize these equations about the trim state \( X_0 \) and trim input \( U_0 \) satisfying \( f(X_0, U_0) = 0 \) to obtain the following,

\[ \dot{x} = A_p x + B_p u + D \]  

(54)

where,

\[
\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -P_0 & R_0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]  

(55)

\[
D = [0 \ 0 \ -d \ 0 \ 0]^T
\]  

(56)

\[
P_0 = \frac{mV}{T}(\Delta x \cos \alpha + \Delta z \sin \alpha)
\]  

(57)

\[
d = \frac{mg}{T}(-\Delta x \sin \alpha + \Delta z \cos \alpha)
\]  

(59)

B. Effect of CG shift on dynamics

With the CG moved, the nonlinear equations get changed to

\[
E(X)\dot{X} = f(X, U) + w(X)
\]  

(52)

\[
w(X) = -mg(\Delta x \cos \alpha + \Delta z \sin \alpha)
\]  

(53)

Table I shows the effect of CG shift, \( \Delta x \) on the trim condition. As it can be easily seen, small changes in CG do not bring about a drastic change in the trim condition. Thus, we make a reasonable assumption that trim conditions remain the same for small CG shifts. We further assume that the change in moment of inertia is small, \( I = I_{yy} - m(\Delta x^2 + \Delta z^2) \sim I_{yy} \). It can be easily shown that CG uncertainty manifests itself in the linear system (49) as,

\[ \dot{x} = \Lambda A_p x + \Lambda B_p u + D \]  

(54)

VI. CONTROL DESIGN FOR HSV

We now discuss the application of the controller developed in section III to the linearized HSV model described in section V subjected to uncertain aerodynamic and CG movements. The linear model in (54) takes the form (1) with \( \Lambda A_p(\lambda) = A, \ \Lambda B_p = B \).

The \( D \) term has been neglected as we consider CG movements only along the \( x \) axis (\( \Delta x \)) and the angle of attack \( \alpha \) is small. Choosing \( B_m = B_p \), it was found that a choice of \( \Psi^* = B_p^{-1}\Lambda B_p \) satisfied assumption (20) for a large class of CG perturbations in \( P \) and \( R \) in (57) and (58).

The goal of the control design is that the states \( X_g = [V, h]^T \) follow a commanded trajectory \( X_{gc} = [V_c, h_c]^T \) while the remaining states \( X_p = [\alpha, \theta, Q]^T \) remain bounded.
This command tracking task is converted into successive regulation around a family of trim points in the following manner.

A family of trim points $X_{g,i}, X_{p,i}, U_i, i = 1, 2, ..., N$ is obtained in the $V - h$ space (see figure 2) such that they solve the nonlinear equation (48)

$$f(X_{g,i}, X_{p,i}, U_i) = 0, \quad i = 1, 2, ..., N$$

(61)

$$X_{p,i} = X_{p,i}(X_{g,i}), \quad U_i = U_i(X_{g,i}).$$

(62)

Using these trim points, a scheduled trim trajectory is obtained by interpolation as,

$$U_0(t) = U_i + M(X_{g,i}, X_{g,c}(t))$$

(63)

$$X_{p0}(t) = X_{p,i} + M(X_{p,i}, X_{g,i}(t))$$

(64)

$$X_{g0}(t) = X_{g,c}(0)$$

(65)

where $M$ is an interpolation function. It can be shown that,

$$\|f(X_{p0}(t), X_{g0}(t), U_0(t))\| \leq \epsilon,$$

where $\epsilon$ is arbitrary small [16]. Thus between two time intervals close to each other, the tracking problem becomes a regulation problem about a given trim point, making the controller designs developed in section III applicable. Though the tracking task has been solved by regulating the plant about the trim trajectories, it was found that gain-scheduling was not needed as the system matrices did not vary significantly with velocity and altitude.

The adaptive control architecture is shown in fig 3. The control input $u(t)$ is a combination of a linear controller augmented with an adaptive component and can be represented as,

$$u(t) = u_{base} + u_{adp}$$

(66)

A. Baseline Controller

The baseline controller is designed using the LQR approach leading to a linear state feedback,

$$J = \int (x^T R_{xx} x + u^T R_{uu} u + \dot{x}^T S \dot{x}) dt$$

(67)

$$u_{base} = -K_x.$$  

(68)

The derivative of states have been penalized to gain control over vehicle acceleration and hence the vehicle load factor. The weighting matrices were chosen in an iterative manner that led to a minimum $||K||_2$.

B. Adaptive Controller

The second component is the adaptive controller and is chosen as,

$$u_{adp} = \theta x$$

(69)

$$\dot{\theta} = -\Gamma e^T P B_m x^T, \quad \Gamma = \gamma \Gamma_0, \gamma > 0$$

(70)

where $\Gamma_0$ satisfies (20). Since the uncertainties in the HSV can be such that the plant is open loop unstable, another feature is added to the adaptive controller in the form of nonlinear damping. Traditionally employed for the control of higher relative degree plants [11], this addition introduces a term $\dot{\theta} x$ into the adaptive control input. As a result, the controller in (69) is modified as,

$$u_{adp} = \theta x + K_D \dot{\theta} x,$$

(71)

where the derivative gain, $K_D$ is a diagonal positive definite matrix. The stability of the resulting system with the derivative term can be easily shown using (28) as the Lyapunov function whose time derivative is,

$$\dot{V} = -e^T Q e - (e^T x)(e^T P B_m)^T K D M (e^T P B_m) \leq 0$$

(72)

The control input with linear component and the adaptive augmentation is,

$$u(t) = -K x + \theta x + K_D \dot{\theta} x$$

(73)

C. Simulation Study

Simulations have been performed on the HSV by trimming the vehicle at $h = 85\ 000$ ft and Mach 8. At the start of the simulation ($t = 0$), the CG is statically shifted by $\Delta x$ and the HSV is commanded a desired trajectory in velocity and altitude. This experiment is more demanding than a slowly time varying dynamic CG movement. The HSV has a reference chord (mean aerodynamic chord) of 17 ft. The aim of the control design is to accommodate forward and backward CG shifts close to 10% of the reference chord. Figures 4-7 compare the performance of linear and adaptive controllers for a forward CG shift of $\Delta x = 1$ ft while tracking a commanded trajectory in the $V - h$ space. It was found that without adaptive augmentation, the linear controller was unable to stabilize the HSV even for small CG shifts. The adaptive controller on the other hand could track the reference trajectory for CG shifts of $-1 < \Delta x < 1.5$ ft. As a negative CG shift (CG moving backwards), makes the closed loop plant with linear feedback unstable, adding non-linear damping to the system was observed to be very helpful.

VII. Summary

This paper proposes an adaptive controller for systems with partially known input matrix $B$. When the input matrix is completely unknown, current adaptive control methods yield controllers that are only locally stable. The controller that is proposed is shown to yield global asymptotic stability when additional information regarding the structure of $B$ is known. The assumptions that we make on the structure of $B$ are in fact quite general and are applicable to a
A rate feedback of the adaptive parameter is added to provide nonlinear damping and hence improve the performance of HSV. Simulation studies performed on a full scale nonlinear model of the HSV show that the adaptive controller can tolerate larger CG shifts as compared to a fixed gain controller while tracking reference commands in velocity and altitude.

REFERENCES

[7] D. Sigthorson, P. Jankovsky, A. Serrani, S. Yurkovich, M. Bolender, and D. Doman, Robust linear output feedback control of an airbreathing hypersonic vehicle, Journal of Guidance,